AD-A264 955

(Classification)

DEFENSE LOGISTICS STU[DIES INFORMATION EXCHANGE]

DTIC ELECTE MAY 25 1993 S C D

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	...NTRACT NUMBER N/A	3 ACCESSION NUMBER
...ERFORMING ORGANIZATION REPRESENTATIVE Robert C. Bilikam, Jr.		5 DISTRIBUTION AVAILABILITY OF REPORT
...ERFORMING ORGANIZATION Operations Research Office	6a OFFICE SYMBOL DESC-RO	6b ADDRESS (City, State, and Zip Code) 1507 Wilmington Pike Dayton, OH 45444
...PONSORING/FUNDING ORGANIZATION Defense Logistics Agency	7a OFFICE SYMBOL DLA-OSP	7b ADDRESS (City, State, and Zip Code) Cameron Station Alexandria, VA 22304-6100
...LSIE SEARCH NUMBER	8a SEARCH DATE (YYMMDD)	8b SEARCH RESULTS ☐ IN PROGRESS ☐ NOT FOUND

...TITLE (Include Security Classification if Applicable)

EOQ for Decreasing Demand Items (UPDATE)

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...TYPE OF REPORT	11a START DATE 10 May 91 EXP COMPLETION DATE	11b DATE OF REPORT (YYMMDD) N/A	11c PAGE COUNT N/A

... ABSTRACT (Continue on reverse if necessary and identify by block number)

An EOQ was developed theoretically for items with a long term decreasing demand scenario. A revised cost equation was developed to find the reorder point and EOQ for the decreasing demand trend.

...CONCLUSIONS (Continue on reverse if necessary and identify by block number)

A decreasing demand EOQ has been developed. The DoD VSL model has been incorporated into the EOQ model.

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... RECOMMENDATIONS (Continue on reverse if necessary and identify by block number)

The study is continuing with applying the EOQ on a data base of items with long term (5-10 year) decreasing demand trends.

...IMPLEMENTING INSTRUCTIONS (Continue on reverse if necessary and identify by ...

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DD Form 1498 (Modified)

(Classification)

8-28-91

EOQ BASED ON DECREASING DEMAND

(Negative Exponential Model)

1. CONCEPTS: The "Economic Order Quantity" (EOQ) is normally based on the quantity that minimizes the average annual combined costs of holding inventory and ordering in the steady state case. However, the decreasing demand scenario means there is no steady state situation. The solution to getting a common criterion function for all different length cycles (procurement cycle periods) is to use the expenditures over unit time as the criterion function. The expenditures in the next cycle due to a buy are calculated and divided by the cycle time. The holding cost for average inventory is applied to the average inventory in the cycle period. Instead of the holding rate per year in the steady state case, the rate per cycle length is used. There is, of course only one order per cycle.

2. ILLUSTRATION: Wilson EOQ.

$E(x) = E(t_2 - t_1)$ = expenditures in the cycle period divided by the cycle period

D = quarter's demand (constant in this case).
Q = buy quantity.
H = holding rate per year.
X = procurement cycleperiod.
= $t_2 - t_1$. t_1 = order receipt time in quarters.
t_2 = order used up.
A = order cost.
Q/2 = "average inventory."
C = unit cost.
H(Q) = holding cost over procurement cycle.
$= \frac{H(t_2-t_1)}{4}$, time in quarters.

$$E(x) = \frac{A + [\text{Average inventory over Cycle}] \cdot H(Q) \cdot C}{x}$$

$$E(x) = \frac{A + \left[\frac{Q}{2}\right] \frac{QHC}{4D}}{Q/D}$$

x = Q/D in this case.

$$E(x) = \frac{A + \left(\frac{Q}{2}\right) \cdot \frac{QHC}{4D}}{Q/D}$$

$$= \frac{AD}{Q} + \left[\frac{Q}{2}\right] \cdot \frac{HC}{4}$$

E(x) In this case is the Normal "average annual cost" fromula for the Wilson EOQ divided by 4 since quarters were used.

3. <u>AVERAGE INVENTORY:</u> The average inventory over the procurement cycle period is used in this formulation:

$$I_{ave} = \frac{\int_{t_1}^{t_2} Q - \begin{bmatrix}\text{Demand}\\ \text{rate}\end{bmatrix} \cdot (t-t_1)\, dt}{t_2-t_1}$$

$$I_{ave} = \frac{\int_{t_1}^{t_2} Q\, dt}{t_2-t_1} - \frac{\int_{t_1}^{t_2} \begin{bmatrix}\text{demand}\\ \text{rate}\end{bmatrix} \cdot (t-t_1)\, dt}{t_2-t_1}$$

$$= Q - \frac{\int_{t_1}^{t_2} \begin{bmatrix}\text{Demand}\\ \text{rate}\end{bmatrix} \cdot (t-t_1)}{t_2-t_1}$$

$$\begin{matrix}\text{Demand}\\ \text{Rate}\end{matrix} = \frac{Q}{t_2-t_1} \quad \text{for Wilson Model}$$

$$I_{ave} = Q - \frac{Q}{(t_2-t_1)^2} \int_{t_1}^{t_2} (t-t_1)\, dt$$

$$= Q - \frac{Q}{(t_2-t_1)^2} \left\{ \left[\frac{t^2}{2}\right]_{t_1}^{t_2} - t_1 \left[t\right]_{t_1}^{t_2} \right\}$$

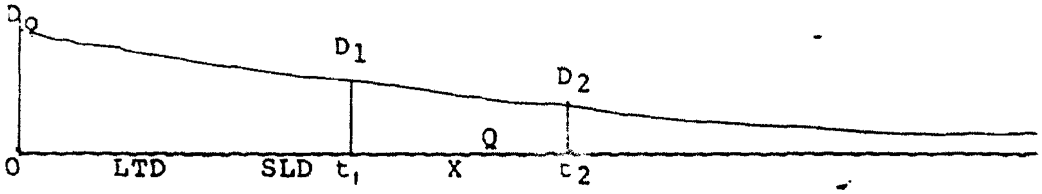
$$= Q - \frac{Q}{(t_2-t_1)^2} \left\{ \frac{t_2^2}{2} - \frac{t_1^2}{2} - t_1 t_2 + t_1^2 \right\}$$

$$= Q - \frac{Q}{(t_2-t_1)^2} \left\{ \frac{1}{2}\left(t_2^2 - 2t_1 t_2 + t_1^2\right) \right\}$$

$$= Q - \frac{Q}{2} = \frac{Q}{2}.$$

4. Negative Exponential Model.

4.1 Fundamentals



ALTD = Administrative Lead Time Days.

PLTD = Procurement Lead Time Days.

SLD = Safety Level Days.

LTD = Lead Time Days. (= ALTD + PLTD)

D_o = Demand rate when ordering (now)

D_1 = Demand rate when order (arrives)

D_2 = Demand rate when order quantity is exhausted.

$D_1 = D_o e^{-a\,(LTD\ +\ SLD)/91.25}$

$D_2 = D_o e^{-a\frac{(LTD\ +\ SLD+X)}{91.25}}$ where $X = t_2 - t_1$, the procurement cycle period.

$\therefore D_2 = D_o e^{-a\,\frac{(LTD+SLD)}{91.25}}\, e^{-ax}$

$= D_1\, e^{-ax}$, $a>o$.

(a) = Decay constant for demand.

$K_1 = e^{-a\frac{(LTD\ +\ SLD)}{91.25}}$

$D_1 = D_o K_1$

$D_2 = D_o K_1 e^{-ax}$

ROP = Re-order Point $\frac{D_0 - D_1}{a}$

4.2 Average Inventory

$$I_{ave} = Q - \frac{\int_{t_1}^{t_2} D_0 e^{-at}(t - t_1)\,dt}{t_2 - t_1}$$

$$Q = \frac{D_1 - D_2}{a} = \frac{D_0 K_1}{a}\left(1 - e^{-ax}\right)$$

$$\int_{t_1}^{t_2} D_0 e^{-at}(t-t_1)\,dt = D_{ave}\cdot(t_2-t_1).$$

$$= D_0\int_{t_1}^{t_2} e^{-at}\,t\,dt - D_0 t_1 \int_{t_1}^{t_2} e^{-at}\,dt$$

$$= -\frac{D_0}{a^2}(e^{-at})(1+at)\Big|_{t_1}^{t_2} - \frac{D_0 t_1}{-a}(e^{-at})\Big|_{t_1}^{t_2}$$

$$= -\frac{D_0}{a^2}(e^{-at_2})(1+at_2) + \frac{D_0(e^{-at_1})}{a^2}(1+at_1)$$

$$+ \frac{D_0 t_1}{a}(e^{-at_2}) - \frac{D_0 t_1}{a}(e^{-at_1})$$

$$= -\frac{D_0 e^{-at_2}}{a^2} - \frac{D_0 t_2 e^{-at_2}}{a} + \frac{D_0 e^{-at_1}}{a^2} + \cancel{\frac{D_0 t_1 e^{-a}}{a}}$$

$$+ \frac{D_0 t_1}{a}(e^{-at_2}) - \cancel{\frac{D_0 t_1}{a}(e^{-at_1})}$$

$$= \frac{D_0}{a^2}\left(e^{-at_1} - e^{-at_2}\right) - (t_2-t_1)\frac{D_0}{a}e^{-at_2}.$$

$$D_{ave} = \frac{D_0}{a^2}\,\frac{(e^{-at_1} - e^{-at_2})}{(t_2-t_1)} - \frac{D_0 e^{-at_2}}{a}.$$

$$I_{ave} = Q - D_{ave} \qquad ; \quad t_2 - t_1 = x.$$

$$= \frac{D_0 K_1}{a}\left[1 - \cancel{e^{-ax}} - \frac{1}{ax} + \frac{e^{-ax}}{ax} + \cancel{e^{-ax}}\right]$$

$$= \frac{D_0 K_1}{a}\left[1 - \frac{1}{ax} + \frac{e^{-ax}}{ax}\right].$$

4.3 Solution for Optimum Procurement Cycle Period.

Let x = Procurement Cycle Period.

E(x) = Expenditures per unit time over x.

$$E(x) = \frac{A + \begin{bmatrix}\text{Average Inventory} \\ \text{Over Cycle}\end{bmatrix} \cdot \frac{XHC}{4}}{x}$$

For the exponential case

$$E(x) = \frac{A + \frac{D_0 K_1}{a}\left[1 - \frac{1}{ax} + \frac{e^{-ax}}{ax}\right] \frac{xHC}{4}}{x}$$

$$E(x) = \frac{A}{x} + \frac{D_0 K_1 HC}{4a}\left[1 - \frac{1}{ax} + \frac{e^{-ax}}{ax}\right]$$

$$\frac{\partial E(x)}{\partial x} = -\frac{A}{x^2} + \frac{D_0 K_1 HC}{4a}\left[\frac{1}{ax^2} - \frac{ae^{-ax}}{ax} - \frac{e^{-ax}}{ax^2}\right]$$

$$\frac{\partial E(x)}{\partial x} = 0 = -\frac{A}{x^2} + \frac{D_0 K_1 HC}{4a^2x^2} + \frac{D_0 K_1 HC}{4a}\left[e^{-ax}\right]\left[-\frac{1}{x} - \frac{1}{ax^2}\right]$$

$$\frac{A - D_0 K_1 HC/4a^2}{x^2} = \left[e^{-ax}\right]\left(\frac{D_0 K_1 HC}{4a^2}\right)\left[-\frac{a}{x} - \frac{1}{x^2}\right]$$

$$\frac{A - D_0 K_1 HC/4a^2}{D_0 K_1 HC/4a^2} = \left[e^{-ax}\right]\left[-ax - 1\right]$$

$$\frac{(D_0 K_1 HC/4a^2) - A}{D_0 K_1 HC/4a^2} = \left[e^{-ax}\right]\left[ax + 1\right].$$

$$D_1 = D_0 K_1.$$

Therefore, the optimum procurement cycle x satisifies the relationship:

$$\frac{(D_1 HC/4a^2) - A}{D_1 HC/4a^2} = \left[e^{-ax}\right]\left[ax + 1\right].$$

$$Q = \frac{D_1}{a}\left[1 - e^{-ax}\right].$$

4.4 CALCULATIONS

The function $y=e^{-ax}(ax + 1)$ can be tabulated for any value of a.

When the constant $\frac{(D_1HC/4a^2)-A}{(D_1HC/4a^2)}$

is equal to Y, the x value corresponds to the optimum procurement cycle.

Notice that Y > o for x > o. The constant term can be less than zero (for $D_1HC/4a^2 < A$), in which case there is no proper solution for x.

Calculations using a Basic Program shows that the curve E(x) and the solution for x approach the Wilson EOQ procurement cycle closely for $a \leq .01$.

4.5 COMPARISON OF CALCULATIONS.

Figure 1a shows the Wilson EOQ derived for a quarterly demand of 100 units at a unit cost of $10 each, with a holding rate per year of .17 and an order cost of $90. The Economic Order Quantity is 6 months or 2 quarters supply (205 units from the EOQ formula). The expenditures per unit time is lowest at this procurement quantity.

Figure 1b shows the same item except demand is decreasing (a=.01). However, since the rate of decrease is very gradual, the expenditures per unit time are about the same as Figure 1a (Wilson EOQ model). The optimum procurement quantity is six months or two quarters of the <u>reduced</u> demand rate which is computed at 190 units instead of 205 units.

Figure 1c shows a drastic reduction in demand rate over time (a=.2). Even though the optimum procurement quantity is 12 months or 4 quarters, the reduced demand rate at the point of receipt dictates that a smaller quantity of 127 units be bought.

Figures 2a, 2b, and 2c are the corresponding results for a unit cost of $100 instead of $10. The Wilson EOQ of Figure 2a indicates an EOQ of 65 units which is 2 months or .67 quarters supply. The corresponding optimum procurement quantity for Figure 2b (a=.01.) gives the same period but a quantity of 64 units due to decreasing demand. Figure 2c (a=.2) shows an increased period of 3 months or 1 quarter but a greatly reduced quantity of 42 units because of the decreased demand rate.

5.6 EQUIVALENT MODELS

Equivalent values of a and m to have the same procurement cycle period (x) can be calculated.

$$Q_{EXP} = \frac{D_1 - D_2}{a} = \frac{D_0 e^{-at_1}}{a}\left(1 - e^{-ax}\right), \quad a > 0.$$

$$Q_{LINEAR} = \left(\frac{D_1 + D_2}{2}\right)x = \frac{2(D_0 + t_1 m) + mx}{2}x$$
$$= (D_0 + t_1 m)x + \frac{mx^2}{2}.$$

Accept the optimum x from the linear model:

$$\frac{D_0 e^{-at_1}\left(1 - e^{-ax}\right)}{a} = (D_0 + t_1 m)x + \frac{mx^2}{2}$$

$$\frac{e^{-at_1}\left(1 - e^{-ax}\right)}{a} = \left(\frac{D_0 + t_1 m}{D_0}\right)x + \frac{mx^2}{2D_0}$$

Consider the right side as a constant. The left side can be tabulated for different values of (a) to determine the equivalent value of (a) for a given value of m. An approximate value for the equivalent (a) can be obtained from

$$a\ (\text{APPROX}) = -\frac{\ln\left(D_1/D_0\right)}{t_1},$$

$$D_1 = D_0 + t_1 m, \quad m \le 0.$$

An example is given in Figure 4a. The item has the characteristics shown in Figure 1b except that a negative linear slope (m) of -5 units / qtr. was applied. The optimum was 7 mo. procurement cycle or Q = 175 units. The equivalent (a) for a negative exponential model is calculated in Figure 4b. The approximate a = .05512 was varied in the left side of the equation in Column 2 (f) until the value of f approximated the right side value of 1.7496. The best fit was obtained at a=.0577118. Using this (a) in Figure 4c resulted in the same procurement cycle (7 mo.) and Q = 175 units in the exponential model.

The slope used can be no more negative than $\left(\frac{1}{t_1 + 12}\right)(-D_0) = \frac{-100}{15.8} = -6.3$ units

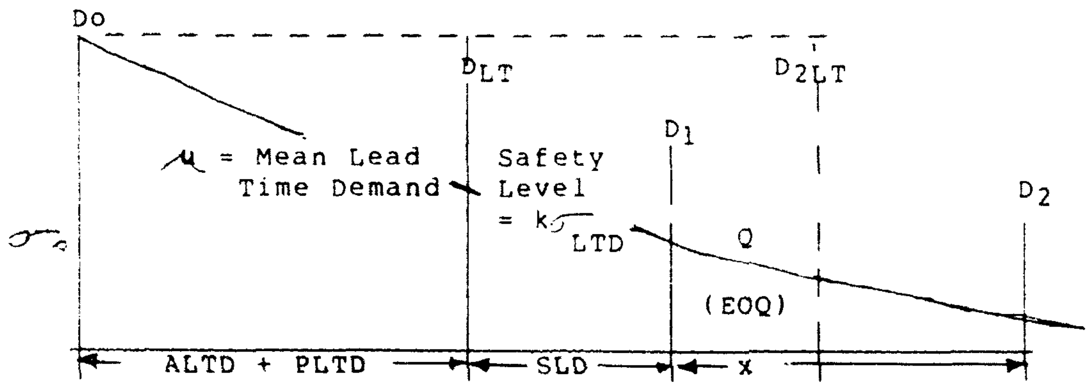
for correct calculation of E(x) at 36 months.

6.0 Variable Safety Level and Decreasing DEMAND EOQ.

6.1 DISCUSSION

The decreasing demand Economic Order Quantity (EOQ) was developed for the negative exponential demand decrease. This was a deterministic model where demand decreased according to a preset curve. The safety level days (SLD) was assumed to be developed from other considerations and used in the formulation. Below the safety level model is developed using probabilistic arguments, with the <u>mean</u> demand at any time following the negative exponential curve.

6.2 FUNDAMENTALS



AL TD = Mean Administrative Lead Time Days

PLTD = Mean Procurement Lead Time Days

SLD = Safety Level Days

D_o = Mean Demand rate (now)

D_{LT} = Mean Demand rate at end of lead time

D_1 = Demand Rate at end of Reorder Point (ROP) period

D_2 = Demand Rate after end of Procurement Cycle

x = Procurement Cycle period (quarters)

ROP = Reorder Point quantity

$= \mu + k\sigma_{LTD}$

σ_{LTD} = Standard Deviation of lead time demand

$$= \sigma_o \cdot \left[\frac{\mu}{(D_o)\cdot\left(\frac{ALTD + PLTD}{91.25}\right)} \right]$$

WILSON EOQ

ORDER COST = 90   HOLDING RATE/YR = 90

QRLY FCST DEM = 100   UNIT COST = 10   ADV = 4000

PCP (QTRS)	ORDER COST	HOLDING COST	EXPEND/TIME
.3333334	90	2.361112	277.0834
.6666667	90	9.444446	149.1667
1	90	21.25	111.25
1.333333	90	37.77778	95.83334
1.666667	90	59.02777	89.41666
2	90	85	87.5 ← EOQ.
2.333333	90	115.6944	88.15476
2.666667	90	151.1111	90.41667
3	90	191.25	93.75
3.333333	90	236.1111	97.83333
3.666667	90	285.6945	102.4621
4	90	340	107.5
4.333334	90	399.0278	112.8526
4.666667	90	462.7778	118.4524
5	90	531.25	124.25
5.333334	90	604.4445	130.2083
5.666667	90	682.361	136.299
6	90	765	142.5
6.333334	90	852.3612	148.7939
6.666667	90	944.4443	155.1667
[illegible]	90	1041.25	161.6072
.333334	90	1142.778	168.1061
.666667	90	1249.028	174.6558
[illegible]	90	1360	181.25
.333333	90	1475.694	187.8833
.666667	90	1596.111	194.5513
[illegible]	90	1721.25	201.25
.333333	90	1851.111	207.9762
.666667	90	1985.695	214.727
0	90	2125	221.5
0.33333	90	2269.027	228.293
0.66667	90	2417.778	235.1042
1	90	2571.25	241.9318
1.33333	90	2729.444	248.7745
1.66667	90	2892.362	255.631
2	90	3060	262.5

CONOMIC ORDER QTY = 205.7983

QFD=100.
C=10.

Fig 1a

WILSON EOQ
ORDER COST = 90 HOLDING RATE/YR = 90
QTRLY FCST DEM = 100 UNIT COST = 100 ADV = 40000

PCP(QTRS)	ORDER COST	HOLDING COST	EXPEND/TIME
.333334	90	23.61111	340.8334
.6666667	90	94.44445	276.6667 ← EOQ.
1	90	212.5	302.5
1.333333	90	377.7778	350.8334
1.666667	90	590.2778	408.1667
2	90	850	470
2.333333	90	1156.944	534.4048
2.666667	90	1511.111	600.4167
3	90	1912.5	667.5
3.333333	90	2361.111	735.3333
3.666667	90	2856.945	803.7122
4	90	3400	872.5
4.333334	90	3990.278	941.6027
4.666667	90	4627.778	1010.952
5	90	5312.5	1080.5
5.333334	90	6044.445	1150.208
5.666667	90	6823.611	1220.049
6	90	7650	1290
6.333334	90	8523.611	1360.044
6.666667	90	9444.444	1430.167
7	90	10412.5	1500.357
7.333334	90	11427.78	1570.606
7.666667	90	12490.28	1640.906
8	90	13600	1711.25
8.333333	90	14756.94	1781.633
8.666667	90	15961.11	1852.052
9	90	17212.5	1922.5
9.333333	90	18511.11	1992.976
9.666667	90	19856.95	2063.477
10	90	21250	2134
10.33333	90	22690.28	2204.543
10.66667	90	24177.78	2275.104
11	90	25712.5	2345.682
11.33333	90	27294.44	2416.275
11.66667	90	28923.61	2486.881
12	90	30600	2557.5

ECONOMIC ORDER QTY = 65.07914

QFD = 100.
C = 100.

Fig 2a

DECREASING DEMAND EOQ USING NEG EXPONENTIAL DEMAND
DECAY CNST = .01 INIT DEMAND = 100 UNIT COST = 10
ALT DAYS = 90 PLT DAYS = 200 SL days = 60
[redacted] DEM AT ROP = 96.23701 ADV = 3849.481 ROP QTY = 376.2985
ORDER COST = 90 HOLDING RATE/YR = .17
FUNCTION FOR DETERMINING EOQ

Fig 1b

X(MO.s)	Y=FUNCTION	DIFF FROM CONST(ABS)
1	.9999945	2.145767E-04
2	.9999779	1.97947E-04
3	.9999504	1.704097E-04
4	.9999119	1.319051E-04
5	.9998626	8.267165E-05 ← OPT = 6 mo.
6	.9998026	2.270937E-05
7	.9997319	4.804135E-05
8	.9996507	1.292825E-04
9	.9995589	2.210736E-04
10	.9994566	3.232956E-04
11	.999344	4.359484E-04
12	.9992209	5.589724E-04
13	.9990878	6.921888E-04
14	.9989443	8.355975E-04
15	.9987909	9.890795E-04
16	.9986273	1.152635E-03
17	.9984538	1.326203E-03
18	.9982703	1.509607E-03
19	.9980771	1.702786E-03
20	.9978741	1.905859E-03
21	.9976614	2.118528E-03
22	.997439	2.340913E-03
23	.9972072	2.572775E-03
24	.9969658	2.814174E-03
25	.9967148	3.065109E-03
26	.9964546	3.325343E-03
27	.996185	3.594935E-03
28	.9959063	3.873646E-03
29	.9956183	4.161656E-03
30	.9953212	4.458666E-03
31	.995015	4.764915E-03
32	.9947	5.079925E-03
33	.9943759	5.403996E-03
34	.9940431	5.736888E-03
35	.9937012	6.078661E-03
36	.9933509	6.429017E-03

CONSTANT = .9997799 OPTIMUM PROC CYCLE PERIOD (MO) = 6

X(QTRs)	ORDER COST	HOLD COST	E(X)=EXPEND/TIME	Q=ORDER QTY
.3333334	90	2.288351	276.8651	32.02591
.6666667	90	9.070191	148.6053	63.94501
1	90	20.40793	110.4079	95.75806
1.333333	90	36.15595	94.61696	127.4651
1.666667	90	56.48065	87.88839	159.0653
2	90	81.25726	85.62863	190.5617 ← OPT = 2 QTRS
2.333333	90	110.4545	85.90909	221.9544
2.666667	90	144.1245	87.79668	253.241
3	90	182.1735	90.72451	284.4238
3.333333	90	224.7265	94.41793	315.5029
3.666667	90	271.6272	98.6256	346.4783
4	90	322.9071	103.2268	377.3514
4.333334	90	378.5166	108.1192	408.1207
4.666667	90	438.5052	113.2511	438.7886
5	90	502.8261	118.5652	469.3527
5.333334	90	571.5052	124.0322	499.8169
5.666667	90	644.4906	129.616	530.179[illegible]
6	90	721.673	135.2788	560.4409
6.333334	90	803.2683	141.0424	590.6021
6.666667	90	889.0503	146.8576	620.662
7	90	979.0736	152.7248	650.6218
7.333334	90	1073.38	158.6427	680.4825
7.666667	90	1171.901	164.5958	710.2433
8	90	1274.622	170.5777	739.9048
8.333333	90	1381.527	176.5832	769.4687
8.666667	90	1492.634	182.6116	798.9335
9	90	1607.897	188.6552	828.3005
9.333333	90	1727.299	194.7106	857.5699
9.666667	90	1850.864	200.779	886.7416
10	90	1978.539	206.8539	915.8165
10.33333	90	2110.31	212.9333	944.7945
10.66667	90	2246.245	219.023	973.6763
11	90	2386.206	225.1097	1002.461
11.33333	90	2530.286	231.2017	1031.151
11.66667	90	2678.359	237.2879	1059.74[illegible]
12	90	2830.5	243.375	1088.245

DECAY CNST = .01 INIT DEMAND = 100 UNIT COST = 100
ALT DAYS = 90 PLT DAYS = 200 SL days = 60
[redacted] DEM AT ROP = 96.23701 ADV = 38494.81 ROP QTY = 376.2985
ORDER COST = 90 HOLDING RATE/YR = .17
FUNCTION FOR DETERMINING EOQ

X (MO.s)	Y=FUNCTION	DIFF FROM CONST(ABS)	
1	.9999945	1.651049E-05	
2	.9999779	1.192093E-07	← OPT = 2 mo
3	.9999504	2.765656E-05	
4	.9999119	6.616116E-05	
5	.9998626	1.153946E-04	
6	.9998026	1.753569E-04	
7	.9997319	2.461076E-04	
8	.9996507	3.273487E-04	
9	.9995589	4.191399E-04	
10	.9994566	5.213619E-04	
11	.999344	6.340146E-04	
12	.9992209	7.570386E-04	
13	.9990878	8.90255E-04	
14	.9989443	1.033664E-03	
15	.9987909	1.187146E-03	
16	.9986273	1.350701E-03	
17	.9984538	1.52427E-03	
18	.9982703	1.707673E-03	
19	.9980771	1.900852E-03	
20	.9978741	2.103925E-03	
21	.9976614	2.316594E-03	
22	.997439	2.538979E-03	
23	.9972072	2.770841E-03	
24	.9969658	3.01224E-03	
25	.9967148	3.263176E-03	
26	.9964546	3.523409E-03	
27	.996185	3.793001E-03	
28	.9959063	4.071713E-03	
29	.9956183	4.359722E-03	
30	.9953212	4.656732E-03	
31	.995015	4.962981E-03	
32	.9947	5.277992E-03	
33	.9943759	5.602062E-03	
34	.9940431	5.934954E-03	
35	.9937012	6.276727E-03	
36	.9933509	6.627083E-03	

Fig 2b

CONSTANT = .999978 OPTIMUM PROC CYCLE PERIOD (MO) = 2

X(QTRs)	ORDER COST	HOLD COST	E(X)=EXPEND/TIME	Q=ORDER QTY	
.3333334	90	22.88351	338.6505	32.02591	
.6666667	90	90.70191	271.0529	63.94501	← OPT = 2/3 QTR.
1	90	204.0793	294.0793	95.75806	
1.333333	90	361.5595	338.6696	127.4651	
1.666667	90	564.8065	392.8839	159.0653	
2	90	812.5726	451.2863	190.5617	
2.333333	90	1104.545	511.948	221.9544	
2.666667	90	1441.245	574.2168	253.241	
3	90	1821.735	637.2451	284.4238	
3.333333	90	2247.265	701.1793	315.5029	
3.666667	90	2716.273	765.3471	346.4783	
4	90	3229.071	829.7678	377.3514	
4.333334	90	3785.166	894.2691	408.1207	
4.666667	90	4385.052	958.9397	438.7886	
5	90	5028.261	1023.652	469.3527	
5.333334	90	5715.052	1088.447	499.8169	
5.666667	90	6444.906	1153.219	530.1796	
6	90	7216.73	1217.788	560.4409	
6.333334	90	8032.683	1282.529	590.6021	
6.666667	90	8890.503	1347.076	620.662	
7	90	9790.736	1411.534	650.6218	
7.333334	90	10733.8	1475.972	680.4825	
7.666667	90	11719.01	1540.306	710.2433	
8	90	12746.22	1604.527	739.9048	
8.333333	90	13815.27	1668.632	769.4687	
8.666667	90	14926.34	1732.655	798.9335	
9	90	16078.97	1796.552	828.3005	
9.333333	90	17272.99	1860.32	857.5699	
9.666667	90	18508.64	1923.997	886.7416	
10	90	19785.39	1987.539	915.8165	
10.33333	90	21103.1	2050.946	944.7945	
10.66667	90	22462.45	2114.292	973.6763	
11	90	23862.06	2177.46	1002.461	
11.33333	90	25302.85	2240.546	1031.151	
11.66667	90	26783.59	2303.451	1059.746	
12	90	28305	2366.25	1088.245	

DECREASING DEMAND EOQ USING NEG EXPONENTIAL DEMAND
DECAY CNST = .2 INIT DEMAND = 100 UNIT COST = 10
ALT DAYS = 90 PLT DAYS = 200 SL days = 60
[redacted] DEM AT ROP = 46.43469 ADV = 1857.388 ROP QTY = 267.8266
ORDER COST = 90 HOLDING RATE/YR = .17
FUNCTION FOR DETERMINING EOQ

X(MO.s)	Y=FUNCTION	DIFF FROM CONST(ABS)
1	.9978742	.1802936
2	.9918631	.1742824
3	.9824769	.1648963
4	.9701759	.1525952
5	.9553751	.1377944
6	.938448	.1208674
7	.9197307	.10215
8	.8995241	8.194351E-02
9	.8780986	6.051797E-02
10	.8556952	3.811455E-02
11	.8325292	1.494855E-02
12	.8087922	8.788466E-03 ← OPT 12 mo.
13	.784654	3.292662E-02
14	.7602655	5.731523E-02
15	.7357589	8.182174E-02
16	.7112511	.1063296
17	.6868443	.1307364
18	.6626273	.1549534
19	.6386771	.1789036
20	.6150601	.2025206
21	.5918328	.2257479
22	.5690432	.2485375
23	.5467316	.2708491
24	.524931	.2926497
25	.5036683	.3139124
26	.4829649	.3346158
27	.4628369	.3547438
28	.4432964	.3742843
29	.4243512	.3932294
30	.4060059	.4115748
31	.3882618	.4293188
32	.3711177	.446463
33	.3545701	.4630106
34	.3386133	.4789674
35	.3232399	.4943408
36	.3084411	.5091396

Fig 1c

CONSTANT = .8175806 OPTIMUM PROC CYCLE PERIOD (MO) = 12

X(QTRs)	ORDER COST	HOLD COST	E(X)=EXPEND/TIME	Q=ORDER QTY
.3333334	90	1.072425	273.2173	14.97356
.6666667	90	4.196944	141.2954	28.98144
1	90	9.241157	99.24116	42.08588
1.333333	90	16.08135	79.56101	54.34521
1.666667	90	24.60163	68.76098	65.81388
2	90	34.6937	62.34685	76.54293
2.333333	90	46.25615	58.3955	86.58
2.666667	90	59.19414	55.9478	95.96978
3	90	73.41897	54.47299	104.7539
3.333333	90	88.84766	53.6543	112.9716
3.666667	90	105.4026	53.29162	120.6593
4	90	123.0111	53.25278	127.8512 ← OPT = 4 QTRS
4.333334	90	141.6052	53.44736	134.5793
4.666667	90	161.1214	53.81173	140.8734
5	90	181.5002	54.30004	146.7616
5.333334	90	202.6859	54.87862	152.2701
5.666667	90	224.6266	55.52234	157.4233
6	90	247.2735	56.21225	162.2441
6.333334	90	270.5811	56.93386	166.7541
6.666667	90	294.5067	57.67601	170.9732
7	90	319.0107	58.43009	174.9202
7.333334	90	344.0555	59.18938	178.6126
7.666667	90	369.6063	59.94865	182.0669
8	90	395.6305	60.70382	185.2984
8.333333	90	422.0977	61.45172	188.3216
8.666667	90	448.9792	62.1899	191.1497
9	90	476.2482	62.91646	193.7954
9.333333	90	503.8798	63.62998	196.2706
9.666667	90	531.8506	64.32938	198.5861
10	90	560.1388	65.01388	200.7522
10.33333	90	588.7238	65.68295	202.7786
10.66667	90	617.5866	66.33625	204.6744
11	90	646.7091	66.97356	206.4479
11.33333	90	676.0747	67.59483	208.107
11.66667	90	705.6678	68.2001	209.6591
12	90	735.4735	68.78946	211.1112

DECREASING DEMAND EOQ USING NEG EXPONENTIAL DEMAND

DECAY CNST = .2 INIT DEMAND = 100 UNIT COST = 100

ALT DAYS = 90 FLT DAYS = 200 SL days = 60

DEM AT ROP = 46.43469 ADV = 18573.88 ROP QTY = 267.8266

ORDER COST = 90 HOLDING RATE/YR = .17

FUNCTION FOR DETERMINING EOQ

X(MO.s)	Y=FUNCTION	DIFF FROM CONST(ABS)	
1	.9978742	1.611614E-02	
2	.9918631	1.010501E-02	
3	.9824769	7.188916E-04	← OPT = 3 mo.
4	.9701759	.0115822	
5	.9553751	2.638298E-02	
6	.938448	4.331005E-02	
7	.9197307	.0620274	
8	.8995241	8.223391E-02	
9	.8780986	.1036595	
10	.8556952	.1260629	
11	.8325292	.1492289	
12	.8087922	.1729659	
13	.784654	.197104	
14	.7602655	.2214927	
15	.7357589	.2459992	
16	.7112511	.270507	
17	.6868443	.2949138	
18	.6626273	.3191309	
19	.6386771	.343081	
20	.6150601	.366698	
21	.5918328	.3899253	
22	.5690432	.4127149	
23	.5467316	.4350265	
24	.524931	.4568271	
25	.5036683	.4780898	
26	.4829649	.4987932	
27	.4628369	.5189212	
28	.4432964	.5384617	
29	.4243512	.5574068	
30	.4060059	.5757522	
31	.3882618	.5934962	
32	.3711177	.6106404	
33	.3545701	.627188	
34	.3386133	.6431448	
35	.3232399	.6585182	
36	.3084411	.673317	

Fig 2c

CONSTANT = .9817581 OPTIMUM PROC CYCLE PERIOD (MO) = 3

X(QTRs)	ORDER COST	HOLD COST	E(X)=EXPEND/TIME	Q=ORDER QTY	
.3333334	90	10.72425	302.1727	14.97356	
.6666667	90	41.96945	197.9542	28.98144	
1	90	92.41157	182.4116	42.08588	← OPT = 1 QTR.
1.333333	90	160.8135	188.1101	54.34521	
1.666667	90	246.0163	201.6098	65.81388	
2	90	346.937	218.4685	76.54293	
2.333333	90	462.5615	236.8121	86.58	
2.666667	90	591.9415	255.728	95.96978	
3	90	734.1897	274.7299	104.7539	
3.333333	90	888.4767	293.543	112.9716	
3.666667	90	1054.026	312.0071	120.6593	
4	90	1230.111	330.0278	127.8512	
4.333334	90	1416.052	347.5505	134.5793	
4.666667	90	1611.214	364.5459	140.8734	
5	90	1815.002	381.0003	146.7616	
5.333334	90	2026.86	396.9111	152.2701	
5.666667	90	2246.266	412.2822	157.4233	
6	90	2472.735	427.1225	162.2441	
6.333334	90	2705.811	441.4438	166.7541	
6.666667	90	2945.067	455.2601	170.9732	
7	90	3190.107	468.5867	174.9202	
7.333334	90	3440.555	481.4393	178.6126	
7.666667	90	3696.063	493.8344	182.0669	
8	90	3956.305	505.7882	185.2984	
8.333333	90	4220.977	517.3172	188.3216	
8.666667	90	4489.791	528.4374	191.1497	
9	90	4762.482	539.1646	193.7954	
9.333333	90	5038.798	549.5141	196.2706	
9.666667	90	5318.507	559.5007	198.5861	
10	90	5601.388	569.1388	200.7522	
10.33333	90	5887.239	578.4425	202.7786	
10.66667	90	6175.866	587.4249	204.6744	
11	90	6467.092	596.0992	206.4479	
11.33333	90	6760.747	604.4777	208.107	
11.66667	90	7056.678	612.5724	209.6591	
12	90	7354.735	620.3946	211.1112	

σ_0 = Standard Deviation of lead time demand at beginning (estimated)

a = Decay constant

H = Holding rate for inventory

C = Unit price (acquisition cost)

6.3 BACKORDER MODEL

The model of Presutti and Trepp in "More Ado About Economic Order Quantities (EOQ)", (Reference 1) is acceptable and can be considered usuable as long as the situation in para 6.2 is referenced. The backorders at a given time

$$B_T = \frac{.5\,\sigma^2}{2\;Q}\,[1 - \exp(-\sqrt{2}\,Q/\sigma)]\,\exp[(-\sqrt{2}k)$$

will be utilized with the exception that $\sigma = \sigma_{LTD}$ as described in para. 6.2. and the "Q" is the decreasing demand EOQ.

6.4 MODEL IV VARIABLE SAFETY LEVEL

Model IV of Reference 1 can be modified to suit the situation of decreasing demand by using the cycle cost concept for the objective function subject to a backorder constraint as discussed in para.6.3 above

$$\text{"Cycle Cost"} = E(Q) = \frac{A + \frac{D_1}{a}\left[1 - \frac{1}{ax} + \frac{e^{-ax}}{ax}\right]\frac{xHC}{4}}{x}$$

Where A = order cost.

D_1 = Demand rate at end of ROP period.

$$Q = \frac{D_1}{a}\left[1 - e^{-ax}\right]$$

Note $\frac{(\dot{D}_{LT} - D_1)}{a} = k\sigma_{LTD}$ (area calculations).

Once k has been determined, D_1 can be found from

$$D_1 = D_{LT} - a\,k\sigma_{LTD}.$$

$$E(x) = \frac{A}{x} + \frac{D_1 HC}{4a} - \frac{D_1\,HC\,(1 - e^{-ax})}{4a^2\,x} = \frac{A}{x} + \frac{D_1 HC}{4a} - \frac{QHC}{4ax}$$

Assume that x and Q are independent of $k.\sigma_{LTD}$. This will simplify the objective function manipulation substantially.

Objective Function K:

Minimize $\sum_{i=1}^{n} E_i(x_i)$ subject to $\sum_{i=1}^{n} B_{Ti} \leq \beta.$
$k_i \sigma_i$

The sum of the "cycle costs" of items in the inventory is to be minimized subject to a constraint on the number of backorders produced using the resulting safety levels.

Proceeding as Model IV of Reference 1:

$$K = \sum_i \left(\frac{A_i}{x_i} + \frac{D_i H C_i}{4 a_i} - \frac{Q_i H C_i}{4 a_i x_i} \right) - \lambda \left[\sum_i \left\{ \frac{.5}{2} \frac{\sigma_i^2 z_i}{Q_i} \left[1 - \exp\left(-\frac{\sqrt{2} Q_i}{\sigma_i} \right) \right] \exp(-\sqrt{2} k \right. - \beta \right] = 0.$$

z_i = essentially weight.

$$\frac{\partial K}{\partial k_i} = \frac{\partial D_i}{\partial k_i} \frac{H C_i}{4 a_i} - \lambda (-\sqrt{2}) \frac{.5}{2} \frac{\sigma_i^2 z_i}{Q_i} \left[1 - \exp\left(-\frac{\sqrt{2} Q_i}{\sigma_i} \right) \right] \exp(-\sqrt{2} k = 0.$$

$$\frac{\partial K}{\partial \lambda} = \sum_i \left[\frac{.5}{2} \frac{z_i \sigma_i^2}{Q_i} \left[1 - \exp\left(-\frac{\sqrt{2} Q_i}{\sigma_i} \right) \right] \exp(-\sqrt{2} k_i) \right] - \beta =$$

Using the second equation which makes the summation to the left equal to β and:

$$\frac{\partial D_i}{\partial k_i} = \frac{\partial}{\partial k_i} \left(D_{LT_i} - a_i k_i \sigma_i \right) = -a_i \sigma_i ;$$

$$\frac{\partial K}{\partial k_i} = -\frac{H C_i \sigma_i}{4} + \lambda (\sqrt{2}) \frac{.5}{2} \left[1 - \exp\left(\frac{-\sqrt{2} Q_i}{\sigma_i} \right) \right] \exp(-\sqrt{2} k_i = 0.$$

Summing over all n relations directly above:

$$-\sum \frac{HC_i \sigma_i}{4} + \lambda(\sqrt{2})\beta = 0.$$

Therefore -

$$-\lambda = \frac{\sum HC_i \sigma_i}{4\sqrt{2}\,\beta} \qquad \text{or} \qquad -4\lambda = \frac{\sum HC_i \sigma_i}{\sqrt{2}\,\beta}$$

Solving each $\frac{\partial K}{\partial k_i} = 0$,

$$k_i = -\frac{1}{\sqrt{2}} \ln \left[\frac{\sqrt{2}\, Q_i\, HC_i}{(-4\lambda)(.5)\,\sigma_i\, z_i \left(1 - \exp\left(\frac{-\sqrt{2}\, Q_i}{\sigma_i}\right)\right)} \right]$$

The safety level $K_i\sigma_i$ has the same results as Model IV of reference.1,

$-\lambda$ is the shadow price of a backorder per quarter, if Z_i is used as 1/(average requisition size).

6.5 PROCEDURES FOR CALCULATING THE EOQ AND SAFETY LEVEL

The procedure based upon the above results for calculating the decreasing demand EOQ and associated variable safety level (using Model IV) is as follows:

(1) Compute x (procurement cycle) and Q (the EOQ quantity from the deterministic model with SLD = o:

$$Q^* = \frac{D_0 k_1^* (1 - e^{-ax^*})}{a}, \quad \left(k_1^* = e^{-a\left(\frac{ALTD + PLTD}{91.25}\right)}\right)$$

x^* = Optimun procurement cycle in quarters, calculated from minimizing $E(x)$ with SLD = o.

(2) Using the value of Q^* and K_1^*) calculate

$$\sigma_i = \sigma_{LTD} = \sigma_0 \left[\frac{D_0 (1 - k_1^*)/a}{D_0 \left(\frac{ALTD + PLTD}{91.25}\right)}\right]$$

Calculate k_i in the usual fashion using Model IV.

Note the constraints:

a. $o \leq k_i \leq 3$

b. Safety Level $\leq k_i \sigma_i \leq \frac{D_1^* - D_{2LT}}{a}$

(Three standard deviation and Lead Time Quantity constraints).
$D_1^* = k_1^* D_0$, $D_{2LT} = D_0 e^{-2a (ALTD + PLTD)/91.25}$

(3) Calculate ROP

$$ROP = \left[D_0 (1 - k_1^*)/a\right] + \text{SAFETY LEVEL}$$

(4) Recalculate Q, using safety level (SLQ)

$$D_1 = D_{LT} - a \cdot SLQ = D_0 k_1^* - a \cdot SLQ$$

$$SLD = \frac{91.25}{a}\left[\ln\left(\frac{D_0}{D_1}\right)\right] - ALTD - PLTD$$

Go back to step 1 for recalculation of Q, using $Q = \frac{D_1(1-e^{-ax})}{a}$

x = Optimum using ALTD+PLTD+SLD = $\frac{91.25\left[\ln\left(\frac{D_0}{D_1}\right)\right]}{a}$

6.6 EXAMPLE:

The item for which Q and SLQ are to be calculated has the characteristics shown below:

ACQ COST	$ 1.00	ALTD	140
HLD RATE	0.17	PLTD	200
ORDER CL	95	SLD	0
ORDER C H	490	LTD+SLD	340
THRESH LB	$25,000	TLTD	340

The safety level quantity (SLQ) is calculated using the first cut at the EOQ (Q) of 8023.7 with safety level days (SLD) set = (o):

SAFETY LEVEL CALCULATIONS:

QFD = 4,054

EOQ = 8,023.7 (2 QTRS)

ADJ SMAD = 2,489.73 (SMAD = 2,500.0)

ADJ LEAD TIME DEMAND QTY = 14,920

RECURRING DEMAND 4 QTRS = 16,000

RECURRING REQUIS. 4 QTRS = 100

ANRDP = 0.0 AVE REQ. SIZE = 160

ESSENTIALITY = 2 (SMCC C)

BACKORDER TARGET = 69,000

SYSTEM CONSTANT = $158,154,229

VSL QTY = 10,068 (226.6 DAYS CURRENT QFD)

ENHANCEMENT = 0 AT 94% (WSAIC = G).

THE EOQ (Q) is now recalculated with the safety level days adjusted to 229 days considering the decreasing demand situation:

$$SLD = \frac{91.25}{a}\left[\ln\left(\frac{D_0}{D_1}\right)\right] - ALTD - PLTD$$

SECOND CALCULATION OF EOQ (Q):

SLD ADJUSTED = 229.13 DAYS

LTD + SLD = 569 DAYS

$K_1{}^* = .991791$

$K_1 = .9863$

$D_1 = 3998.5$

EOQ = Q = 7979.3, x = 2 QTRS

ROP = 25105.8

SOR = 33085.2

6.7 REFERENCE

Presutti, V. and Trepp, R. "More Ado About Economic Order Quantities (EOQ)," Naval Research Logistics Quarterly, June 1970 (Vol 17, No. 2).

$$D_{AVE} \cdot (t_2 - t_1) = \int_{t_1}^{t_2} [D_1 + m(t - t_1)](t - t_1)\, dt.$$

$$u = t - t_1, \quad du = dt.$$

$$D_{AVE} \cdot (t_2 - t_1) = \int_0^x [D_1 + mu]\, u\, du$$

$$= \left[\frac{D_1 u^2}{2} + \frac{m u^3}{3}\right]_0^x.$$

$$= \frac{D_1 x^2}{2} + \frac{m x^3}{3}.$$

$$I_{AVE} = Q - \frac{\left[\frac{D_1 x^2}{2} + \frac{m x^3}{3}\right]}{x}, \quad Q = \left(\frac{D_1 + D_2}{2}\right) \cdot x.$$

$$= \left[\frac{D_1 + D_2}{2}\right] x - \frac{D_1 x}{2} - \frac{m x^2}{3}.$$

$$= \frac{D_1 + D_1 + m x}{2} x - \frac{D_1 x}{2} - \frac{m x^2}{3}$$

$$= D_1 x + \frac{m x^2}{2} - \frac{D_1 x}{2} - \frac{m x^2}{3}$$

$$= \frac{D_1 x}{2} + \frac{m x^2}{6}. \qquad m = 0, \; I_{AVE} = \frac{D_0 x}{2} = \frac{Q}{2}.$$

$$D_1 = D_0 + m t_1, \quad m \le 0.$$

$$t_1 = \frac{LTD + SLD}{91.25}.$$

5.3 Solution for Optimum Procurement Cycle Period.

$$E(x) = \frac{A + \begin{bmatrix}\text{Average Inventory} \\ \text{Over Cycle}\end{bmatrix} \cdot \frac{XHC}{4}}{x}$$

For the linear case,

$$E(x) = \frac{A + \left[\frac{D_1 x}{2} + \frac{m x^2}{6}\right] \frac{xHC}{4}}{x}$$

$$= \frac{A}{x} + \frac{HC}{4}\left[\frac{D_1 x}{2} + \frac{m x^2}{6}\right]$$

$$\frac{\partial E(x)}{\partial x} = 0 = \frac{-A}{x^2} + \frac{HC}{4}\left[\frac{D_1}{2} + \frac{m x}{3}\right]$$

$$0 = -A + \frac{D_1 HC}{8} x^2 + \frac{m HC}{12} x^3$$

Therefore x is the solution to the cubic equation given above.

$$A = \frac{HC}{4}\left[\frac{D_1 x^2}{2} + \frac{m x^3}{3}\right].$$

5.4 CALCULATIONS

If $y = \frac{HC}{4}\left[\frac{D_1 x^2}{2} + \frac{m x^3}{3}\right]$

and A = the Constant, the optimum x is where y = A.

$Q = \left(\frac{D_1 + D_2}{2}\right) x$.

Let m = o (Wilson EOQ). $D_1 = D_2 = D_o$.

$$\frac{A}{x^2} = \frac{HC}{4}\left[\frac{D_o}{2}\right]. \qquad x^2 = \frac{8A}{HCD_o}$$

$$Q = D_o x = D_o\sqrt{\frac{8A}{HCD_o}} = \sqrt{\frac{8AD_o}{HC}}.$$

The slope can be no more (negative) than $\left(\frac{1}{t_1+t_2}\right)\left(-D_o\right)$ to avoid wrong calculations of the Criterion function at 36 months procurement cycle period.

5.5 COMPARISON OF CALCULATIONS.

The Wilson EOQ is calculated in Figure 3a with the slope m = o, for c = $100 and D_o = 100 units. The same results as Figure 2a are obtained. When the slope m = -1 unit/QTR, the results of Figure 3b are obtained with the optimum at 2 months of reduced demand giving 64 instead of 66 units for the optimum order quantity.

DECREASING DEMAND EOQ USING LINEAR (SLOPED) DEMAND
SLOPE CNST = 0 INIT DEMAND = 100 UNIT COST = 100
ALT DAYS = 90 PLT DAYS = 200 SL days = 60
[redacted] DEM AT ROP = 100 ADV = 40000 ROP QTY = 383.5617
ORDER COST = 90 HOLDING RATE/YR = .17
FUNCTION FOR DETERMINING EOQ

Fig 3a

X(MO.s)	Y=FUNCTION	DIFF FROM CONST(ABS)	
1	23.61111	66.38889	
2	94.44445	4.444451	← OPT = 2 MO.
3	212.5	122.5	
4	377.7778	287.7778	
5	590.2778	500.2778	
6	850	760	
7	1156.944	1066.944	
8	1511.111	1421.111	
9	1912.5	1822.5	
10	2361.111	2271.111	
11	2856.944	2766.944	
12	3400	3310	
13	3990.278	3900.278	
14	4627.778	4537.778	
15	5312.5	5222.5	
16	6044.445	5954.445	
17	6823.611	6733.611	
18	7650	7560	
19	8523.611	8433.611	
20	9444.444	9354.444	
21	10412.5	10322.5	
22	11427.78	11337.78	
23	12490.28	12400.28	
24	13600	13510	
25	14756.94	14666.94	
26	15961.11	15871.11	
27	17212.5	17122.5	
28	18511.11	18421.11	
29	19856.95	19766.95	
30	21250	21160	
31	22690.28	22600.28	
32	24177.78	24087.78	
33	25712.5	25622.5	
34	27294.44	27204.44	
35	28923.61	28833.61	
36	30600	30510	

CONSTANT = 90 OPTIMUM PROC CYCLE PERIOD (MO) = 2

X(QTRs)	ORDER COST	HOLD COST	E(X)=EXPEND/TIME	Q=ORDER QTY	
.3333334	90	23.61111	340.8334	33.33334	
.6666667	90	94.44445	276.6667	66.66667	← OPT = $\frac{2}{3}$ QTR.
1	90	212.5	302.5	100	
1.333333	90	377.7778	350.8334	133.3333	
1.666667	90	590.2777	408.1666	166.6667	
2	90	850	470	200	
2.333333	90	1156.944	534.4048	233.3333	
2.666667	90	1511.111	600.4167	266.6667	
3	90	1912.5	667.5	300	
3.333333	90	2361.111	735.3333	333.3333	
3.666667	90	2856.945	803.7122	366.6667	
4	90	3400	872.5	400	
4.333334	90	3990.278	941.6027	433.3334	
4.666667	90	4627.778	1010.952	466.6667	
5	90	5312.5	1080.5	500	
5.333334	90	6044.445	1150.208	533.3334	
5.666667	90	6823.611	1220.049	566.6666	
6	90	7650	1290	600	
6.333334	90	8523.611	1360.044	633.3334	
6.666667	90	9444.443	1430.167	666.6666	
7	90	10412.5	1500.357	700	
7.333334	90	11427.78	1570.606	733.3334	
7.666667	90	12490.28	1640.906	766.6666	
8	90	13600	1711.25	800	
8.333333	90	14756.94	1781.633	833.3333	
8.666667	90	15961.11	1852.052	866.6667	
9	90	17212.5	1922.5	900	
9.333333	90	18511.11	1992.976	933.3333	
9.666667	90	19856.95	2063.477	966.6667	
10	90	21250	2134	1000	
10.33333	90	22690.28	2204.543	1033.333	
10.66667	90	24177.78	2275.104	1066.667	
11	90	25712.5	2345.682	1100	
11.33333	90	27294.44	2416.275	1133.333	
11.66667	90	28923.62	2486.881	1166.667	
12	90	30600	2557.5	1200	

DECREASING DEMAND EOQ USING LINEAR (SLOPED) DEMAND
LOPE CNST =-1 INIT DEMAND = 100 UNIT COST = 100
LT DAYS = 90 PLT DAYS = 200 SL days = 60
·DEM AT ROP = 96.16438 ADV = 38465.76 ROP QTY = 376.2057
RDER COST = 90 HOLDING RATE/YR = .17
FUNCTION FOR DETERMINING EOQ

Fig 3b

X(MO.s)	Y=FUNCTION	DIFF FROM CONST(ABS)	
.	22.65301	67.34699	
2	90.40217	.4021683	OPT 2 mo
3	202.9326	112.9326	
4	359.9297	269.9297	
5	561.0783	471.0783	
6	806.0639	716.0639	
7	1094.571	1004.571	
8	1426.287	1336.287	
9	1800.894	1710.894	
10	2218.078	2128.078	
11	2677.526	2587.526	
12	3178.922	3088.922	
13	3721.952	3631.952	
14	4306.299	4216.299	
15	4931.65	4841.65	
16	5597.69	5507.69	
17	6304.103	6214.103	
18	7050.575	6960.575	
19	7836.793	7746.793	
20	8662.438	8572.438	
21	9527.199	9437.199	
22	10430.76	10340.76	
23	11372.81	11282.81	
24	12353.02	12263.02	
25	13371.09	13281.09	
26	14426.71	14336.71	
27	15519.54	15429.54	
28	16649.29	16559.29	
29	17815.64	17725.64	
30	19018.26	18928.26	
31	20256.86	20166.86	
32	21531.1	21441.1	
33	22840.68	22750.68	
34	24185.29	24095.29	
35	25564.6	25474.6	
36	26978.3	26888.3	

ONSTANT = 90 OPTIMUM PROC CYCLE PERIOD (MO) = 2

X(QTRs)	ORDER COST	HOLD COST	E(X)=EXPEND/TIME	Q=ORDER QTY	
.3333334	90	22.67925	338.0377	31.99924	
6666667	90	90.61204	270.9181	63.88737	← OPT $\left(\frac{2}{3}\text{QTR}\right)$
.	90	203.641	293.641	95.66438	
..333333	90	361.6087	338.7065	127.3303	
.666667	90	564.3577	392.6146	158.8851	
	90	811.7306	450.8653	190.3288	
.333333	90	1103.57	511.53	221.6613	
.666667	90	1439.719	573.6445	252.8828	
	90	1820.019	636.673	283.9932	
.333333	90	2244.314	700.2941	314.9924	
.666667	90	2712.445	764.3032	345.8805	
	90	3224.256	828.564	376.6575	
.333334	90	3779.589	892.9821	407.3234	
.666667	90	4378.286	957.4898	437.8782	
	90	5020.192	1022.038	468.3219	
.333334	90	5705.146	1086.59	498.6545	
.666667	90	6432.993	1151.116	528.876	
	90	7203.576	1215.596	558.9863	
.333334	90	8016.736	1280.011	588.9856	
.666667	90	8872.314	1344.347	618.8737	
	90	9770.159	1408.594	648.6507	
.333334	90	10710.11	1472.742	678.3165	
.666667	90	11692	1536.783	707.8713	
	90	12715.69	1600.711	737.3151	
.333333	90	13781.01	1664.521	766.6476	
.666667	90	14887.81	1728.208	795.8691	
	90	16035.92	1791.769	824.9795	
.333333	90	17225.19	1855.199	853.9786	
.666667	90	18455.48	1918.497	882.8669	
	90	19726.6	1981.66	911.6438	
.33333	90	21038.41	2044.685	940.3096	
.66667	90	22390.76	2107.571	968.8646	
	90	23783.48	2170.316	997.3082	
.33333	90	25216.41	2232.918	1025.641	
.66667	90	26689.41	2295.378	1053.862	
	90	28202.2	2357.692	1081.973	

SLOPE CNST =-5 INIT DEMAND = 100 UNIT COST = 10
ALT DAYS = 90 FLT DAYS = 200 SL days = 60
[redacted] DEM AT ROF = 80.82191 ADV = 3232.877 ROF QTY = 346.7818
ORDER COST = 90 HOLDING RATE/YR = .17
FUNCTION FOR DETERMINING EOQ

X (MO.s)	Y=FUNCTION	DIFF FROM CONST(ABS)
1	1.882061	88.11794
2	7.423305	82.5767
3	16.46632	73.53368
4	28.85372	61.14629
5	44.42806	45.57194
6	63.03196	26.96804
7	84.50801	5.491997 ← OPT = 7 mo
8	108.6988	18.6988
9	135.4469	45.44692
10	164.595	74.59496
11	195.9855	105.9855
12	229.4612	139.4612
13	264.8646	174.8646
14	302.0382	212.0382
15	340.8248	250.8248
16	381.0668	291.0668
17	422.6069	332.6069
18	465.2877	375.2877
19	508.9517	418.9517
20	553.4415	463.4416
21	598.5999	508.5999
22	644.2692	554.2692
23	690.2922	600.2922
24	736.5115	646.5115
25	782.7695	692.7695
26	828.9088	738.9088
27	874.7722	784.7722
28	920.2022	830.2022
29	965.0414	875.0414
30	1009.132	919.1324
31	1052.318	962.3179
32	1094.44	1004.44
33	1135.342	1045.342
34	1174.866	1084.866
35	1212.855	1122.855
36	1249.151	1159.151

4a

CONSTANT = 90 OPTIMUM PROC CYCLE PERIOD (MO) = 7

X(QTRs)	ORDER COST	HOLD COST	E(X)=EXPEND/TIME	Q=ORDER QTY
.3333334	90	1.895178	275.6855	26.66286
.6666667	90	7.528243	146.2924	52.77017
1	90	16.82049	106.8205	78.32191
1.333333	90	29.69322	89.76991	103.3181
1.666667	90	46.06772	81.64063	127.7587
2	90	65.8653	77.93265	151.6438
2.333333	90	89.00722	76.71738	174.9734 ← OPT = 2 1/3 QTR.
2.666667	90	115.4149	77.03057	197.7473
3	90	145.0094	78.33648	219.9658
3.333333	90	177.7122	80.31367	241.6286
3.666667	90	213.4446	82.75762	262.7359
4	90	252.1278	85.53195	283.2877
4.333334	90	293.6832	88.54228	303.2839
4.666667	90	338.032	91.72114	322.7245
5	90	385.0956	95.01912	341.6096
5.333334	90	434.7953	98.3991	359.9391
5.666667	90	487.0521	101.8327	377.7131
6	90	541.7877	105.2979	394.9315
6.333334	90	598.9231	108.7773	411.5943
6.666667	90	658.3798	112.257	427.7017
7	90	720.0791	115.7256	443.2534
7.333334	90	783.942	119.1739	458.2496
7.666667	90	849.8901	122.5944	472.6903
8	90	917.8448	125.9806	486.5753
8.333333	90	987.7269	129.3272	499.9049
8.666667	90	1059.458	132.6298	512.6789
9	90	1132.96	135.8844	524.8972
9.333333	90	1208.153	139.0878	536.5601
9.666667	90	1284.959	142.2371	547.6675
10	90	1363.299	145.3299	558.2191
10.33333	90	1443.095	148.364	568.2153
10.66667	90	1524.267	151.3376	577.656
11	90	1606.738	154.2489	586.5411
11.33333	90	1690.427	157.0965	594.8706
11.66667	90	1775.258	159.8793	602.6446
12	90	1861.151	162.5959	609.863

	a	$f = \dfrac{e^{-at_1}(1-e^{-ax})}{a}$	$\lvert f - \text{CONST} \rvert$	
1	5.311182E-02	1.790075	4.034102E-02	
2	5.321183E-02	1.789184	3.945089E-02	
3	5.331182E-02	1.788294	3.856027E-02	
4	5.341182E-02	1.787404	3.767002E-02	
5	5.351182E-02	1.786513	3.677976E-02	
6	5.361182E-02	1.785624	3.589082E-02	
7	5.371182E-02	1.784736	3.500223E-02	
8	5.381182E-02	1.783848	3.411484E-02	
9	5.391182E-02	1.78296	3.322685E-02	
10	5.401182E-02	1.782072	3.233898E-02	
11	5.411182E-02	1.781186	3.145242E-02	
12	5.421182E-02	1.7803	.0305661	
13	5.431182E-02	1.779414	2.968097E-02	
14	5.441182E-02	1.778529	2.879512E-02	
15	5.451183E-02	1.777644	2.791035E-02	
16	5.461182E-02	1.776759	2.702522E-02	
17	5.471182E-02	1.775874	2.614033E-02	
18	5.481182E-02	1.774991	2.525735E-02	
19	5.491182E-02	1.774107	2.437365E-02	
20	5.501182E-02	1.773224	2.349067E-02	
21	5.511182E-02	1.772343	2.260924E-02	
22	5.521182E-02	1.77146	2.172673E-02	
23	5.531182E-02	1.770578	2.084494E-02	
24	5.541182E-02	1.769697	1.996374E-02	
25	5.551182E-02	1.768817	.0190835	
26	5.561183E-02	1.767937	1.820362E-02	
27	5.571182E-02	1.767058	1.732421E-02	
28	5.581182E-02	1.766178	.0164448	
29	5.591182E-02	1.765299	1.556575E-02	
30	5.601182E-02	1.764421	.0146879	
31	5.611182E-02	1.763543	1.380944E-02	
32	5.621182E-02	1.762665	1.293194E-02	
33	5.631182E-02	1.761789	1.205587E-02	
34	5.641182E-02	1.760912	1.117814E-02	
35	5.651182E-02	1.760035	1.030159E-02	
36	5.661182E-02	1.759161	9.427071E-03	
37	5.671183E-02	1.758285	8.551001E-03	
38	5.681182E-02	1.75741	7.676125E-03	
39	5.691182E-02	1.756536	6.802321E-03	
40	5.701182E-02	1.755662	5.928159E-03	
41	5.711182E-02	1.754788	5.054951E-03	
42	5.721182E-02	1.753915	4.181981E-03	
43	5.731182E-02	1.753043	3.30925E-03	
44	5.741182E-02	1.752171	2.436996E-03	
45	5.751182E-02	1.751299	1.565695E-03	
46	5.761182E-02	1.750427	6.939173E-04	
47	5.771182E-02	1.749557	1.769066E-04	← BEST FIT.
48	5.781182E-02	1.748687	.0010463	
49	5.791182E-02	1.747817	1.916647E-03	
50	5.801182E-02	1.746946	2.787471E-03	

DECAY CNST = 5.771182E-02 INIT DEMAND = 100 UNIT COST = 10
ALT DAYS = 90 FLT DAYS = 200 SL days = 60
[redacted] DEN AT ROP = 80.14278 ADV = 3205.711 ROP QTY = 344.0755
ORDER COST = 90 HOLDING RATE/YR = .17

FUNCTION FOR DETERMINING EOQ

X(MO.s)	Y=FUNCTION	DIFF FROM CONST(ABS)	
1	.9998172	8.617997E-03	
2	.9992786	8.079291E-03	
3	.9983974	7.198155E-03	
4	.997187	5.987764E-03	
5	.9956602	4.460991E-03	
6	.9938298	2.630532E-03	
7	.9917076	5.083084E-04	← OPT (7 mo.)
8	.9893054	1.893878E-03	
9	.9866348	4.564524E-03	
10	.9837068	7.492483E-03	
11	.9805321	1.066715E-02	
12	.9771212	1.407802E-02	
13	.9734846	1.771468E-02	
14	.9696317	2.156758E-02	
15	.9655721	.0256272	
16	.9613152	2.988398E-02	
17	.9568702	3.432906E-02	
18	.9522454	3.895384E-02	
19	.9474496	4.374963E-02	
20	.9424908	.0487085	
21	.9373771	5.382222E-02	
22	.9321159	5.908335E-02	
23	.926715	6.448424E-02	
24	.9211816	.0700177	
25	.9155225	7.567674E-02	
26	.9097448	8.145451E-02	
27	.9038546	8.734464E-02	
28	.8978589	.0933404	
29	.8917634	9.943587E-02	
30	.8855742	.105625	
31	.8792972	.111902	
32	.8729379	.1182613	
33	.8665018	.1246975	
34	.8599939	.1312054	
35	.8534194	.1377798	
36	.8467834	.1444159	

$\boxed{4c}$

CONSTANT = .9911992 OPTIMUM PROC CYCLE PERIOD (MO) = 7

X(QTRs)	ORDER COST	HOLD COST	E(X)=EXPEND/TIME	Q=ORDER QTY	
.3333334	90	1.879151	275.6375	26.45893	
.6666667	90	7.472327	146.2085	52.41378	
1	90	16.70748	106.7075	77.87409	
1.333333	90	29.51408	89.63556	102.8492	
1.666667	90	45.82495	81.49498	127.3486	
2	90	65.57423	77.78711	151.381	
2.333333	90	88.69532	76.58372	174.9556	← OPT (2 1/3 QTR)
2.666667	90	115.1235	76.92132	198.081	
3	90	144.7969	78.26563	220.7659	
3.333333	90	177.6544	80.29633	243.0185	
3.666667	90	213.6331	82.80903	264.8471	
4	90	252.674	85.66849	286.2598	
4.333334	90	294.7201	88.78156	307.2645	
4.666667	90	339.7144	92.08166	327.869	
5	90	387.5981	95.51962	348.081	
5.333334	90	438.318	99.05962	367.9078	
5.666667	90	491.8209	102.6743	387.3568	
6	90	548.0509	106.3418	406.4353	
6.333334	90	606.96	110.0463	425.1503	
6.666667	90	668.494	113.7741	443.5086	
7	90	732.6043	117.5149	461.5172	
7.333334	90	799.2416	121.2602	479.1827	
7.666667	90	868.3568	125.0031	496.5116	
8	90	939.9041	128.738	513.5102	
8.333333	90	1013.836	132.4604	530.1851	
8.666667	90	1090.108	136.1664	546.5422	
9	90	1168.675	139.8528	562.5876	
9.333333	90	1249.494	143.5172	578.3273	
9.666667	90	1332.521	147.1573	593.7672	
10	90	1417.714	150.7714	608.9128	
10.33333	90	1505.032	154.3579	623.7699	
10.66667	90	1594.435	157.9158	638.3439	
11	90	1685.883	161.4439	652.6402	
11.33333	90	1779.337	164.9415	666.6641	
11.66667	90	1874.759	168.4079	680.4208	
12	90	1972.111	171.8425	693.9154	

7/24/91

Bob:

I'm sorry to take so long to get back to you on the long-term trend EOQ. I think your approach is a good starting point. It basically reduces the EOQ to account for future demand rate. I also agree that this "dampening" should apply to the reorder point period as well.

What about the EOQ computation itself? You propose using the flat-demand EOQ to develop the initial PCP, and then adjusting to account for the future demand rate. But is it possible to derive an optimum EOQ directly? The average inventory will be different from the flat-line model, and will not be constant over successive PCPs. As a matter of fact, successive PCPs may have different lengths. Sounds a little like what we used to call a stage problem, where the decision for some future PCP (stage) depends on decisions made about previous PCPs (stages). I'm a little rusty on that stuff.

All that aside, I would like to see you press on with your proposed approach. I'm curious to see how much we can safely reduce our buy quantities. I appreciate you help.

Thanks

Mike Pory

DEFENSE LOGISTICS AGENCY
CAMERON STATION
ALEXANDRIA, VIRGINIA 22304-6100
DLA-OSP
OFFICIAL BUSINESS

Defense Electronics Supply Center
ATTN: DESC-RO (Bob Billikam/Nanda Balwally)
1507 Wilmington Pike
Dayton, OH 45444-5000

DESC-RO (Mr. Siliken/(AV)986-5917/len)   12 Sep 91

SUBJECT: EOQ for Decreasing Demand

TO: DLA-OSP (Mr. Mike Posy)

1. References:

a. [illegible] DLA-OSP, 24 Aug 91, subject as above.

b. [illegible] Aug 91, subject as above.

2. Here is [illegible] EOQ for Decreasing Demand discussion. [illegible] linear decreasing demand is developed and the relati[illegible] Model is discussed. An equivalent model using [illegible]tial demand is derived giving the same procure[illegible] economic order quantity as a linear decreasing [illegible]

3. Your comments will be appreciated.

3/11KAm

1 Encl

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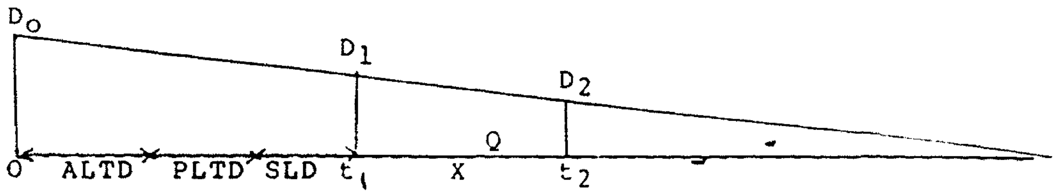
CC:
DLA-LO

5.0 LINEAR DECREASING DEMAND TREND

5.1 FUNDAMENTALS



ALTD = Administrative Lead Time Days.

PLTD = Procurement Lead Time Days.

SLD = Safety Level Days.

LTD = Total Lead Time (Days) = ALTD + PLTD.

D_0 = Demand rate when ordering (now)

D_1 = Demand rate when order (arrives)

D_2 = Demand rate when order quantity is exhausted.

X = Procurement Cycle Period (quarters)
$= t_2 - t_1$

M = Slope in units/qtr. (negative-decreasing) (≤ 0)

$$D_1 = \left(\frac{LTD + SLD}{91.25}\right) \cdot (M) + D_0$$

$$D_2 = D_1 + mx$$

$$D_t = D_0 + mt$$

$$ROP = \frac{(LTD + SLD)}{91.25} \cdot \frac{(D_0 + D_1)}{2}$$

5.2 Average Inventory

$$I_{ave} = \frac{Q - \int_{t_1}^{t_2} \text{Demand rate} \cdot (t - t_1)\, dt}{t_2 - t_1}$$

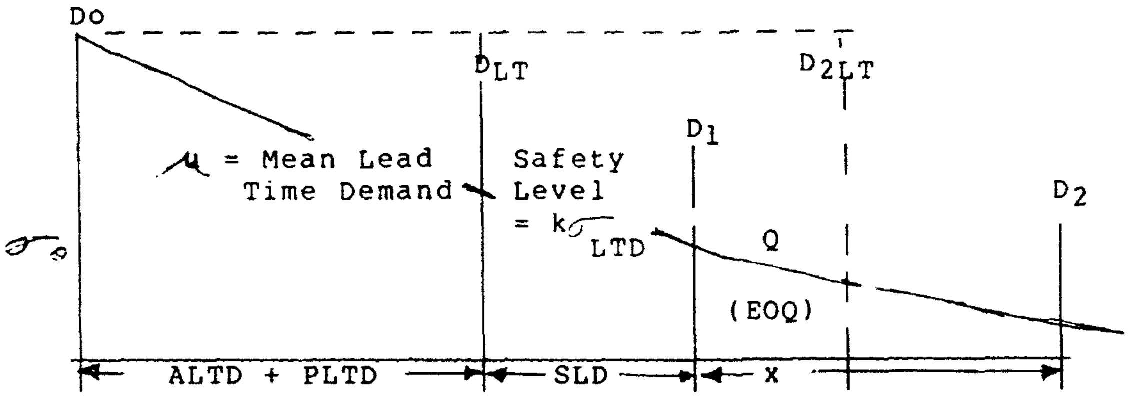
$$= \frac{Q - \int_{t_1}^{t_2} \left[D_1 + m(t - t_1)\right] \cdot (t - t_1)\, dt}{}$$

6.0 Variable Safety Level and Decreasing DEMAND EOQ.

6.1 DISCUSSION

The decreasing demand Economic Order Quantity (EOQ) was developed for the negative exponential demand decrease. This was a deterministic model where demand decreased according to a preset curve. The safety level days (SLD) was assumed to be developed from other considerations and used in the formulation. Below the safety level model is developed using probabilistic arguments, with the mean demand at any time following the negative exponential curve.

6.2 FUNDAMENTALS



AL TD = Mean Administrative Lead Time Days

PLTD = Mean Procurement Lead Time Days

SLD = Safety Level Days

D_o = Mean Demand rate (now)

D_{LT} = Mean Demand rate at end of lead time

D_1 = Demand Rate at end of Reorder Point (ROP) period

D_2 = Demand Rate after end of Procurement Cycle

x = Procurement Cycle period (quarters)

ROP = Reorder Point quantity

$$= \mu + k\sigma_{LTD}$$

σ_{LTD} = Standard Deviation of lead time demand

$$= \sigma_o \cdot \left[\frac{\mu}{(D_o)\cdot\left(\frac{ALTD+PLTD}{91.25}\right)} \right]$$

σ_o = Standard Deviation of lead time demand at beginning (estimated)

a = Decay constant

H = Holding rate for inventory

C = Unit price (acquisition cost)

6.3 BACKORDER MODEL

The model of Presutti and Trepp in "More Ado About Economic Order Quantities (EOQ)", (Reference 1) is acceptable and can be considered usuable as long as the situation in para 6.2 is referenced. The backorders at a given time

$$B_T = \frac{.5\,\sigma^2}{2\;\;Q} \quad [1 - \exp(-\sqrt{2}\,Q/\sigma)]\ \exp[(-\sqrt{2}k)$$

will be utilized with the exception that $\sigma = \sigma_{LTD}$ as described in para. 6.2. and the "Q" is the decreasing demand EOQ.

6.4 MODEL IV VARIABLE SAFETY LEVEL

Model IV of Reference 1 can be modified to suit the situation of decreasing demand by using the cycle cost concept for the objective function subject to a backorder constraint as discussed in para.6.3 above

$$\text{"Cycle Cost"} = E(Q) = \frac{A + \frac{D_1}{a}\left[1 - \frac{1}{ax} + \frac{e^{-ax}}{ax}\right]\frac{xHC}{4}}{x}$$

Where A = order cost.

D_1 = Demand rate at end of ROP period.

$$Q = \frac{D_1}{a}\left[1 - e^{-ax}\right]$$

Note $\frac{(D_{LT} - D_1)}{a} = k\sigma_{LTD}$ (area calculations).

Once k has been determined, D_1 can be found from

$$D_1 = D_{LT} - a\,k\sigma_{LTD}.$$

$$E(x) = \frac{A}{x} + \frac{D_1HC}{4a} - \frac{D_1\,HC\,(1 - e^{-ax})}{4a^2\,x} = \frac{A}{x} + \frac{D_1HC}{4a} - \frac{QHC}{4ax}$$

Assume that x and Q are independent of $k.\sigma_{LTD}$. This will simplify the objective function manipulation substantially.

Objective Function K:

$$\underset{k_i\,\sigma_i}{\text{Minimize}} \quad \sum_{i=1}^{n} F_i(x_i) \text{ subject to } \sum_{i=1}^{n} B_{Ti} \leq \beta.$$

The sum of the "cycle costs" of items in the inventory is to be minimized subject to a constraint on the number of backorders produced using the resulting safety levels.

Proceeding as Model IV of Reference 1:

$$K = \sum_i \left(\frac{A_i}{x_i} + \frac{D_i\,HC_i}{4a_i} - \frac{Q_i\,HC_i}{4a_i x_i} \right) - \lambda \left[\sum_i \left\{ \frac{.5}{2} \frac{\sigma_i^2 z_i}{Q_i} \left[1 - \exp\left(-\frac{\sqrt{2}\,Q_i}{\sigma_i}\right) \right] \exp(-\sqrt{2}k \right. \right.$$
$$\left. - \beta \right] = 0.$$

z_i = essentially weight.

$$\frac{\partial K}{\partial k_i} = \frac{\partial D_i}{\partial k_i} \frac{HC_i}{4a_i} - \lambda(-\sqrt{2}) \frac{.5}{2} \frac{\sigma_i^2 z_i}{Q_i} \left[1 - \exp\left(-\frac{\sqrt{2}\,Q_i}{\sigma_i}\right) \right] \exp(-\sqrt{2}k$$
$$= 0.$$

$$\frac{\partial K}{\partial \lambda} = \sum_i \left[\frac{.5}{2} \frac{z_i \sigma_i^2}{Q_i} \left[1 - \exp\left(-\frac{\sqrt{2}\,Q_i}{\sigma_i}\right) \right] \exp(-\sqrt{2}k_i) \right] - \beta =$$

Using the second equation which makes the summation to the left equal to β and:

$$\frac{\partial D_i}{\partial k_i} = \frac{\partial}{\partial k_i} \left(D_{LT_i} - a_i k_i \sigma_i \right) = -a_i \sigma_i \; ;$$

$$\frac{\partial K}{\partial k_i} = -\frac{HC_i \sigma_i}{4} + \lambda(\sqrt{2}) \frac{.5}{2} \left[1 - \exp\left(\frac{-\sqrt{2}\,Q_i}{\sigma_i}\right) \right] \exp(-\sqrt{2}k_i$$
$$= 0.$$

Summing over all n relations directly above:

$$-\sum \frac{HC_i \sigma_i}{4} + \lambda(\sqrt{2})\beta = 0.$$

Therefore -

$$-\lambda = \frac{\sum HC_i \sigma_i}{4\sqrt{2}\,\beta} \qquad \text{or} \quad -4\lambda = \frac{\sum HC_i \sigma_i}{\sqrt{2}\,\beta}$$

Solving each $\frac{\partial K}{\partial k_i} = 0,$

$$k_i = -\frac{1}{\sqrt{2}} \ln \left[\frac{\sqrt{2}\; Q_i\; HC_i}{(-4\lambda)(.5)\,\sigma_i\, z_i \;\; 1 - \exp\left(\frac{-\sqrt{2}\; Q_i}{\sigma_i}\right)} \right]$$

The safety level $K_i\sigma_i$ has the same results as Model IV of reference.1,

$-\lambda$ is the shadow price of a backorder per quarter, if Z_i is used as 1/(average requisition size).

6.5 PROCEDURES FOR CALCULATING THE EOQ AND SAFETY LEVEL

The procedure based upon the above results for calculating the decreasing demand EOQ and associated variable safety level (using Model IV) is as follows:

(1) Compute x (procurement cycle) and Q (the EOQ quantity from the deterministic model with SLD = o:

$$Q^* = \frac{D_0 k_1^* (1 - e^{-ax^*})}{a}, \quad \left(k_1^* = e^{-a\left(\frac{ALTD + PLTD}{91.25}\right)}\right)$$

x^* = Optimun procurement cycle in quarters, calculated from minimizing E(x) with SLD = o.

(2) Using the value of Q^* and K_1^*) calculate

$$\sigma_i = \sigma_{LTD} = \sigma_0 \left[\frac{D_0 (1 - k_1^*)/a}{D_0 \left(\frac{ALTD - PLTD}{91.25}\right)}\right]$$

Calculate k_i in the usual fashion using Model IV.

Note the constraints:

a. $o \leq k_i \leq 3$

b. Safety Level $\leq k_i \sigma_i \leq \frac{D_1^* - D_{2LT}}{a}$

(Three standard deviation and Load Time Quantity constraints).
$D_1^* = k_1^* D_0$, $D_{2LT} = D_0 e^{-2a(ALTD + PLTD)/91.25}$

(3) Calculate ROP

$$ROP = \left[D_0 (1 - k_1^*)/a\right] + \text{SAFETY LEVEL}$$

(4) Recalculate Q, using safety level (SLQ)

$$D_1 = D_{LT} - a \cdot SLQ = D_0 k_1^* - a \cdot SLQ$$

$$SLD = \frac{91.25}{a}\left[\ln\left(\frac{D_0}{D_1}\right)\right] - ALTD - PLTD$$

Go back to step 1 for recalculation of Q, using $q = \frac{D_1(1-e^{-ax})}{a}$

x = Optimum using ALTD+PLTD+SLD = $\frac{91.25\left[\ln\left(\frac{D_0}{D_1}\right)\right]}{a}$

6.6 EXAMPLE:

The item for which Q and SLQ are to be calculated has the characteristics shown below:

ACQ COST	$ 1.00	ALTD	140
HLD RATE	0.17	PLTD	200
ORDER CL	95	SLD	0
ORDER C H	490	LTD+SLD	340
THRESH LB	$25,000	TLTD	340

The safety level quantity (SLQ) is calculated using the first cut at the EOQ (Q) of 8023.7 with safety level days (SLD) set = (o):

SAFETY LEVEL CALCULATIONS:

QFD = 4,054

EOQ = 8,023.7 (2 QTRS)

ADJ SMAD = 2,489.73 (SMAD = 2,500.0)

ADJ LEAD TIME DEMAND QTY = 14,920

RECURRING DEMAND 4 QTRS = 16,000

RECURRING REQUIS. 4 QTRS = 100

ANRDP = 0.0 AVE REQ. SIZE = 160

ESSENTIALITY = 2 (SMCC C)

BACKORDER TARGET = 69,000

SYSTEM CONSTANT = $158,154,229

VSL QTY = 10,068 (226.6 DAYS CURRENT QFD)

ENHANCEMENT = 0 AT 94% (WSAIC = G).

THE EOQ (Q) is now recalculated with the safety level days adjusted to 229 days considering the decreasing demand situation:

$$SLD = \frac{91.25}{a}\left[\ln\left(\frac{D_0}{D_1}\right)\right] - ALTD - PLTD$$

SECOND CALCULATION OF EOQ (Q):

SLD ADJUSTED = 229.13 DAYS

LTD + SLD = 569 DAYS

$K_1^* = .991791$

$K_1 = .9863$

$D_1 = 3998.5$

EOQ = Q = 7979.3, x = 2 QTRS

ROP = 25105.8

SOR = 33085.2

6.7 REFERENCE

Presutti, V. and Trepp, R. "More Ado About Economic Order Quantities (EOQ)," Naval Research Logistics Quarterly, June 1970 (Vol 17, No. 2).